

# **- Cosmic Nuclear Reactions: Terminology and Questions -**

**NOAJ Astrophysics Lecture Series, November 2013**

**Introduction to Observational Nuclear Astrophysics  
Lecture #3**

**by Roland Diehl**

# Nuclei as Natural Objects

- Atomic Nuclei as Stable Objects
  - ★ Short-Ranged Nuclear Force,  $\sim x \text{ fm} = 10^{-13} \text{ cm}$
- Origin of our Knowledge about Nuclei

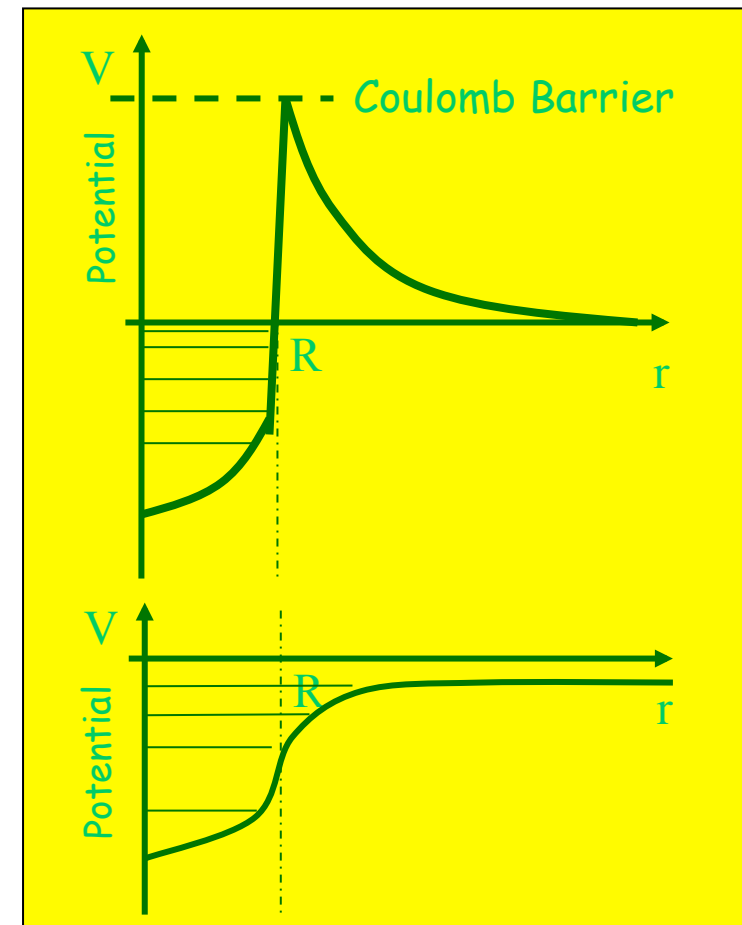
## ★ Existence of Nuclei

- ☞ Milliken, Fraunhofer, Planck -> quantized state multiplicity
- ☞ Rutherford -> compact nucleus, e-
- ☞ Bequerel -> radioactive decay with particle emission

## ★ Constituents

- ☞ Proton
- ☞ Neutron

$$m(Z, N) = Zm_p + Nm_n - Bc^2$$



# Atomic Nuclei: Potential Well for Nucleons

## ★ Nucleons

☞ Neutrons

☞ Protons

## ★ Nuclear Radius

☞ From Rutherford Scattering

-> ~ 1 fm

## ★ Nuclear Density

☞ ~  $10^{13} \text{ g cm}^{-3}$

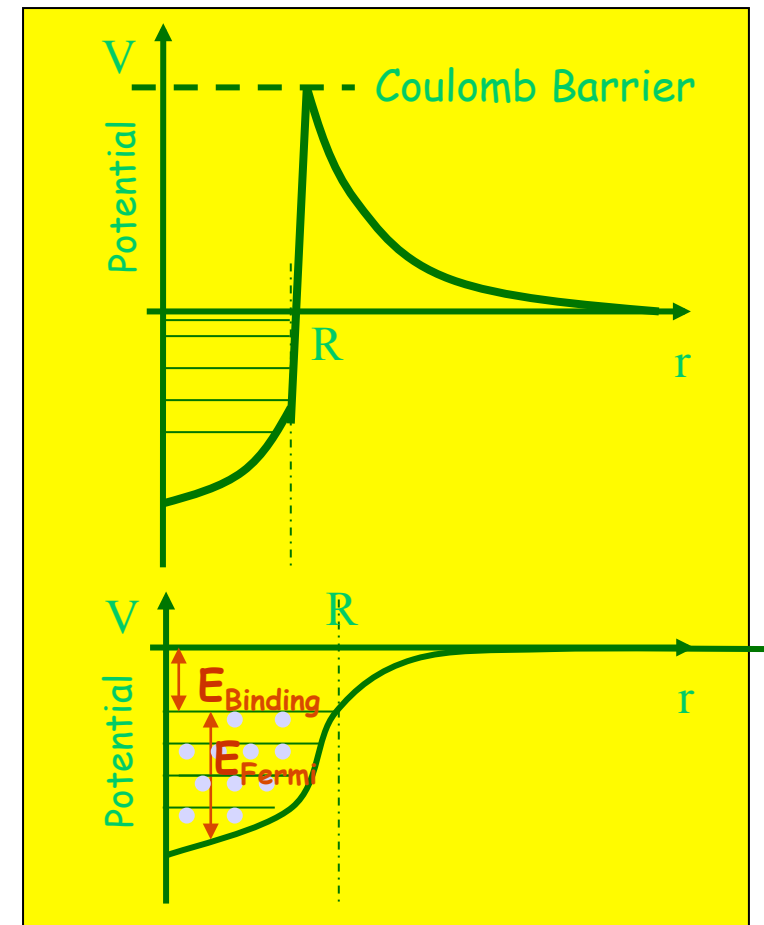
## ★ Number of Nucleonic Levels:

☞ From above, plus

Phase Space Considerations:  $\hbar^3 = V_{\text{state}}$

$$\frac{dn}{dp} = \frac{4\pi p^2}{h^3} V \quad \rightarrow \quad n = \int \frac{4\pi p^2}{h^3} V \cdot dp = \frac{p^3 V}{6\pi^2 \hbar^3}$$

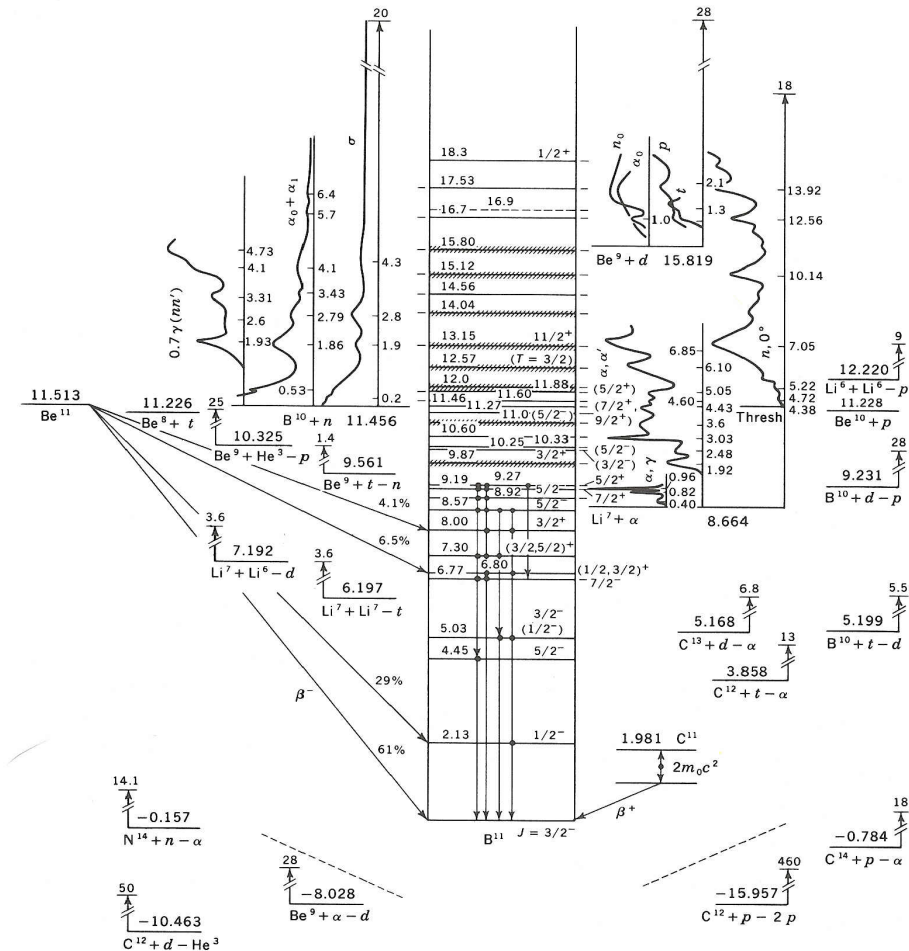
$$E_{\text{fermi}} = \frac{p_f^2}{2m} \approx \frac{250^2}{2 \cdot 938} = 33 [\text{MeV}]$$



# Nuclear Energy Levels: Examples



★ Mostly from  
Nuclear Physics  
Experiments

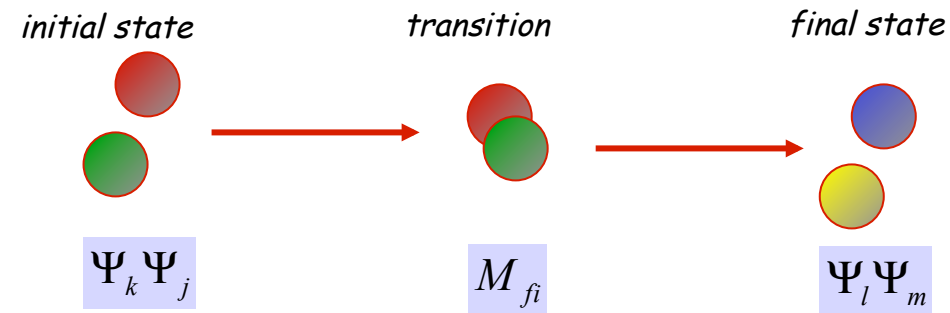


**Fig. 4-13** Energy-level diagram of the  $B^{11}$  nucleus. Energy is plotted vertically on this diagram in units of MeV, with the ground state of  $B^{11}$  taken as the zero of the energy scale. Each known bound state of  $B^{11}$  is labeled by its excitation energy and by the spin and parity of the state. Also shown is the energy required to separate  $B^{11}$  into two particles, for example,  $Li^7 + \alpha$ , and the mass-energy released in the formation of  $B^{11}$  by specific reactions, e.g.,  $Be^9 + He^3 - p$ . This energy-level diagram is typical of those encountered in the nuclear literature, e.g., T. Lauritsen and F. Ajzenberg-Selove, *Energy Levels of Light Nuclei VII, A = 5-10*, *Nucl. Phys.*, **78**:1 (1966).



# Nuclear Reactions: Basic Concepts

## ★ Terminology



## ★ Key Considerations

👉 Initial Situation, Initial States

-> Multiplicity

👉 Impact Parameters, Reaction Geometries

-> Coordinate Choices

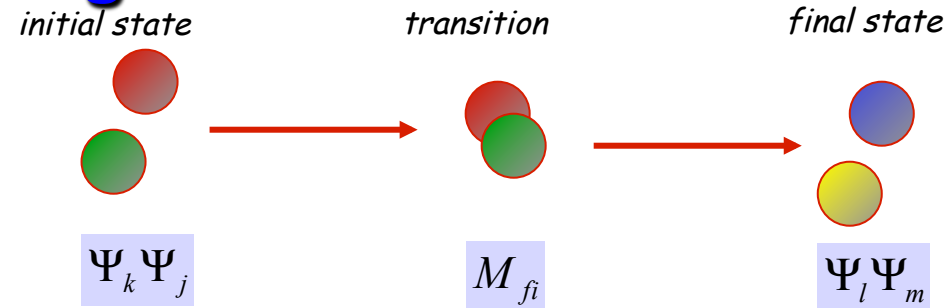
👉 Final States (Phase Space)

-> Multiplicity

👉 Nuclear Transition Probabilities

-> Nuclear Structure & Forces

# Discussing Nuclear Reactions: Viewpoints



## ★ Key Considerations

- |  |                                    |
|--|------------------------------------|
| 👉 Initial Situation, Initial States      | -> Averaging                       |
| 👉 Impact Parameters, Reaction Geometries | -> Appropriate Coordinates         |
| 👉 Final States (Phase Space)             | -> Integration, Statistical Weight |
| 👉 Nuclear Transition Probabilities       | -> Nuclear States                  |

## ★ Different Viewing Points:

### 👉 Geometrical Aspects

- » all Aspects which do NOT Relate to the Type of Reaction and its Physics

### 👉 Types of Nuclear Transitions, Nuclear Structure & Force Impacts

- » all Aspects which Do Relate to the Type of Reaction and its Physics

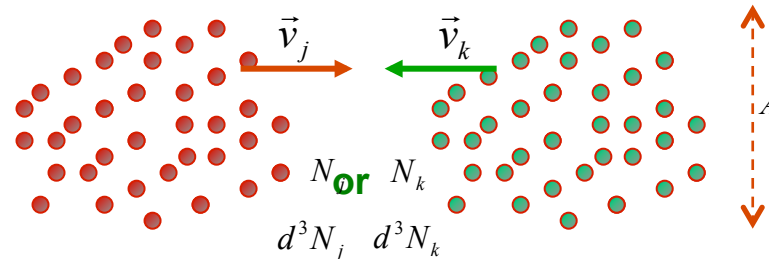
### 👉 State Multiplicities in Entry and Exit Channels: Averaging/Integration

- » all Aspects which Refer to the Phase Space and its Proper Accounting

- Microscopic View
- Alternatively: Systemic/Thermodynamic View

# Terminology: Description of Reactions

- Reactions among Populations of Particles



★ “Reaction Rate”  $R = [\text{Number of Reactions}] / [[\text{Time}][\text{Volume}]]$

☞ i.e.

$$R = \frac{N_{\text{projectiles}} \cdot N_{\text{target}} \cdot P_{\text{transition}}}{C_{\text{norm}}}$$

or, specifically

$$R = \frac{N_j \cdot A v \Delta t \cdot N_k \cdot A v \Delta t \cdot \sigma}{\Delta t \cdot A v \Delta t \cdot A}$$

or, integrating over particle populations:

$$R = \iint \sigma \cdot \overbrace{|\vec{v}_j - \vec{v}_k|} \cdot d^3 N_j \cdot d^3 N_k$$

★ Example: Maxwell-Boltzmann Distribution of Particle Velocities:

»

$$d^3 N = N_0 \left( \frac{m}{2\pi \cdot kT} \right)^{3/2} \cdot e^{-\frac{mv^2}{2kT}} \cdot d^3 v$$

with  $d^3 v = 4\pi \cdot v^2 \cdot dv$   
as the volume element in velocity space

# Terminology: Distribution Functions

- Particle Populations, in center-of-mass Coordinates

$$R_{jk} = \langle \sigma v \rangle \cdot N_j N_k$$

Reaction Rate per Pair of Particles, integrated over Distributions

👉 In Velocity Space:

$$R_{jk} = \int_0^{\infty} N_j N_k \cdot \sigma(v) \cdot \Phi(v) \cdot dv$$

👉 In Energy Space:

$$R_{jk} = \int_0^{\infty} N_j N_k \cdot \left( \frac{2E}{m} \right)^{1/2} \cdot \sigma(E) \cdot \Psi(E) \cdot dE$$

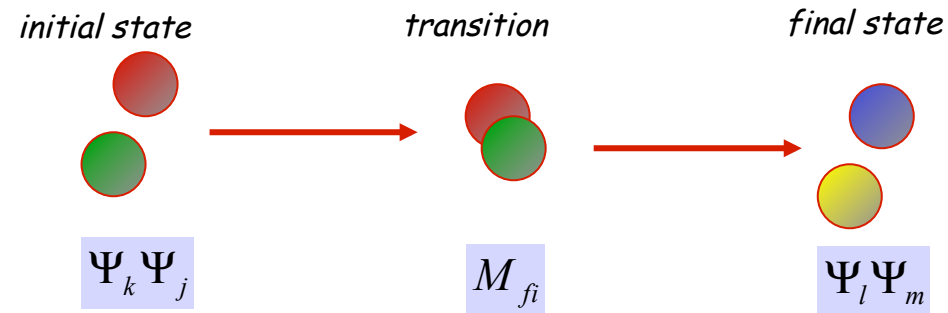
👉 Hence, for a Thermal (= Maxwell-Boltzmann) Population, the Properly-Averaged Reaction Rate per Particle Pair is:

$$\langle \sigma \cdot v \rangle = \left( \frac{8}{\pi \cdot \mu} \right)^{1/2} \cdot \left( \frac{1}{kT} \right)^{3/2} \cdot \int_0^{\infty} E \cdot \sigma(E) \cdot e^{\frac{-E}{kT}} \cdot dE$$

using  $\mu = \frac{M_j M_k}{M_j + M_k}$

as the “reduced mass”

# Comparison: Nuclear Reactions in the Lab



- **Goal of Laboratory Nuclear Physics:**
  - ★ **Measurement of Nuclear Properties**
- **Method**
  - ★ **Carefully Prepare Initial State**
  - ★ **Ensure Validity of Approximations**
    - ☞ **Target and Projectile are Free (solid state energies negligible)**
    - ☞ **Large Energy Transfers**
  - ★ **Carefully Scan / Select Outgoing States**
    - ☞ **Select Secondary Particle Types, Angles, ...**

# Other Views of 'Reaction Rates'

- Other Ways to Characterize Cosmic Nuclear Reactions:

- ★ "Mean Lifetime" of a Particle Against Nuclear Reactions

☞ Destruction of a Species  $k$  by a Reaction  $R_{kj}$

$$R_{jk} = \lambda_k \cdot N_j = \frac{1}{\tau_k} \cdot N_j$$

- ★ Specific Energy Production

☞ Reaction Rate \* specific (density-normalized) Q-Value of Reaction

$$\varepsilon_{jk} = R_{jk} \cdot \frac{Q}{\rho} \quad [\text{erg/sec}]$$

- ★ Normalized Reaction Rates: Relative Fractions of Species

☞ with Mole Fractions ( $N_A$ =Avogadro's Number)

$$N_i = \rho \cdot N_A \cdot \frac{X_i}{A_i} = \rho \cdot N_A \cdot Y_i$$

☞ The fractional reaction rate  $j \rightarrow k$  is

$$\frac{dY_{jk}}{dt} = Y_i \cdot Y_j \cdot \rho \cdot N_A \cdot \langle \sigma \cdot v \rangle \quad \text{with} \quad \dot{\varepsilon}_{jk} = \sum \dot{Y}_i \cdot N_A \cdot \Delta M_i c^2$$

mass excess of species  $i$

# Peculiarities in Cosmic Environments

- Nuclear Reactions in Cosmic Plasma, in Dense Regions

## ★ Screening of Nuclear Charge by Electrons

☞ Reduced Coulomb Repulsion → IMPORTANT “Correction”

☞ Can be Separated in Most Environments:

$$\langle \sigma \cdot v \rangle_{jk} = f_{scr}(Z_j, Z_k, \rho, T, Y_j, Y_k) \cdot \langle \sigma \cdot v \rangle_{jk,0}$$

“Cosmic Rate”

“Laboratory Value”

☞  $f_{scr}$  can be  $10^x$  → “Pycno-Nuclear” Reactions  
e.g. in Neutron Star Crusts

## ★ Note: Screening also Affects Laboratory Measurements

☞ Screening from Atomic and from Additional Free Electrons  
– Reaction Targets as Metals: Debye e- Cloud,  $r_{Debye} < r_{Bohr}$

# Reaction Types: Photon Reactions

## ★ Photon Energy Distribution Function

☞ **Planck Distribution**

$$d^3 N_\gamma = \left( \frac{1}{\pi^2 \cdot (c\hbar)^3} \right) \cdot \frac{E_\gamma^2}{e^{\frac{E_\gamma}{kT}} - 1} \cdot dE_\gamma$$

☞ **Therefore:**

$$R_j = \lambda_{j\gamma}(T) \cdot N_j = \frac{\int d^3 N_j}{\pi^2 c^3 \hbar^3} \cdot \int_0^\infty \frac{c \cdot \sigma(E_\gamma) \cdot E_\gamma^2}{e^{\frac{E_\gamma}{kT}} - 1} dE_\gamma$$

– **i.e.: No Relative Energies**

## ★ Annotation:

☞ **Due to Principle of “Detailed Balance”:**

**Radiative Capture Rate = Photo-Destruction Rate**

$$R_{j\gamma} = R_{lm}$$

-> can eliminate  $s_j(E_g)$  altogether,  
i.e. decay rate (T) given only by nuclear properties (see below)



# Reaction Types: Lepton Reactions; Weak Decay

- Electron Reactions

- ★ Huge Mass Difference

☞  $m_e \sim 1/2000 * m_{\text{nucleus}} \Rightarrow \text{Relative Velocity} = v_e$

☞  $R_j = \lambda_{je}(T, \rho, Y_e) \cdot N_j$  ,  $Y_e = \sum_i Z_i Y_i$  in fully-ionized plasma

- ★ Integration over Applicable Electron Energy Distribution

☞ Thermal Distribution, with  $T_e$

☞ Partially-Degenerate Distributions

☞ Fermi Distribution (fully degenerate)

- Neutrino Reactions

- ★ Analogous to Electron Reactions

☞ Neutrino Thermalization above  $\rho \sim 10^{12} \text{ g/cm}^3$

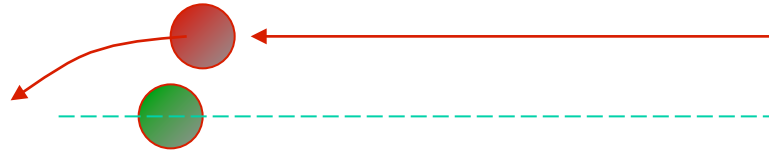
- Weak Decay

- ★ Characterized by Radioactive Lifetime  $\tau$

$$R_j = \lambda_j \cdot N_j = \frac{1}{\tau_j} \cdot N_j$$



# Nuclear Reactions: “Geometrical” Aspect



- “Impact Parameter” -> Centrifugal Barrier for Reactions

★ Plane Wave Approximation  $\Psi = \Psi_0 e^{-ikt}$

☞ Spherical Coordinates

$$\Psi = \varphi(r) \cdot Y_l^m(\theta, \phi)$$

– de Broglie Wavelength

$$r_l = \frac{l \cdot \hbar}{m \cdot v} = l \cdot \tilde{\lambda}_{\text{deBroglie}}$$

★ Reaction Cross Section per Specific Angular-Momentum State  $l$

$$\sigma(l) = \pi(r_{l+1}^2 - r_l^2)$$

$$\sigma(l) = \pi \cdot \tilde{\lambda}^2 \cdot (2l + 1)$$

i.e., in typical units:

$$\sigma(l) = \frac{65.7}{A \cdot E} \left[ \frac{\text{barn}}{\text{a.m.u.} \cdot \text{keV}} \right] \propto \frac{1}{E} \propto \frac{1}{v^2}$$

# Nuclear Reaction Cross Section Estimate



## ★ Order-of-Magnitude Estimation from Size of Nucleus

☞ **Impact Parameter must reach Range of Nuclear Forces**

$$r_j = r_0 A^{1/3} \quad \text{with } r_0 \sim \text{fm} = 10^{-13} \text{cm}$$

$$\sigma_{class} = \pi \cdot r_0^2 \cdot (A_j^{1/3} + A_k^{1/3})^2 \quad \text{where } r_0^2 \sim 4.5 \cdot 10^{-26} \text{ cm}^2 = 45 \text{ mbarn}$$

## ★ However:

- $^{15}\text{N}(\text{p}, \alpha)^{12}\text{C}$ : 0.5 barn
- $^3\text{He}(\alpha, \gamma)^7\text{B}$   $10^{-6}$  barn
- $\text{p}(\text{p}, \text{e}+\text{n})\text{d}$   $10^{-20}$  barn

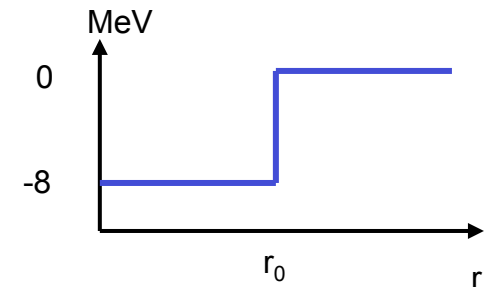
☞ **Estimate from Size of Nucleus only for Strong Reactions!**

# Nuclear Reaction Types: Neutron Capture



★ **Binding Energy of Nucleons in Nucleus: ~7-8 MeV**

☞ ~independent of  $E_n$



★ **Cross Section  $s(E_n)$ :**

☞ **Geometry: Need a Close Encounter ('reactive' collision)**

$$\rightarrow s_{n,geom} \sim 1 / E_n \sim 1 / v_n^2$$

☞ **Phase Space:**

**Entry Channel:**

$$\sim v_n$$

☞ **Exit Channel:**

~ const. (need  $E_n \ll Q$ , -> thermal Neutrons)

$$\sigma_n \propto \frac{1}{v_n^2} \cdot v_n$$

$$\langle \sigma \cdot v_n \rangle \approx const$$

**not dependent on Energy!**

## ★ Relevant Potentials:

### ☞ Nuclear Potential

- $V_0 \sim 7\text{-}8 \text{ MeV}$

### ☞ Coulomb Potential

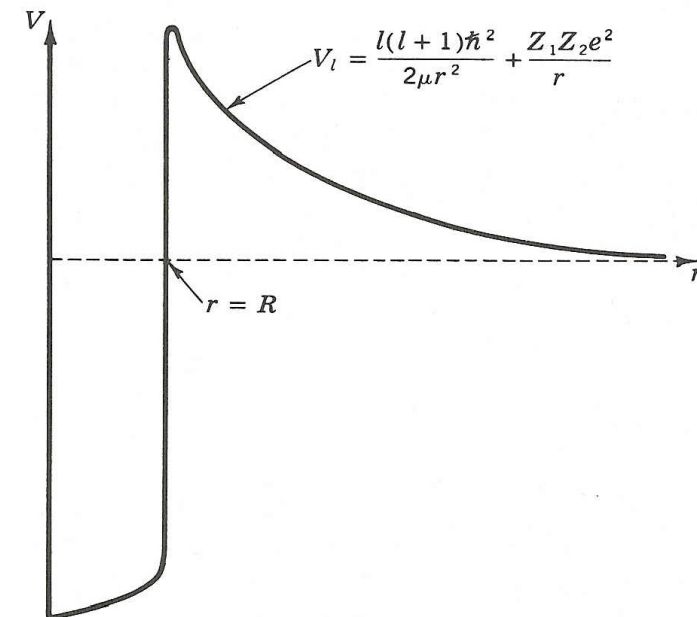
- $V_C = Z_1 Z_2 e^2 / r$

- $E_C \sim 550 \text{ keV}$  for p+p

- » Thermal:  $T = 6.4 \cdot 10^9 \text{ K}$   
 $(T_{\text{sun}} \sim 1.5 \cdot 10^7 \text{ K} \rightarrow E_{\text{th, sun}} \sim 1.9 \text{ keV})$

### ☞ Centrifugal

- $V_Z = l(l+1) \hbar^2 / 2 m r^2$



## ★ “classical turning point”



$$r_{\text{ctp}} = R_0 = Z_1 Z_2 e^2 / E_{\text{kin,p}}$$

# Charged Particle Reactions

★ **Reaction = Tunneling \* Nuclear Transition**

$$P_{\text{reac}} = P_{\text{tunneling}} \cdot P_{\text{nuclear}}$$

☞ **Tunneling a Potential Barrier**

» Landau-Lifshitz...

$$P_T = e^{-\frac{4\pi}{\hbar} \cdot \int_{r_c}^{\infty} \left[ 2m \left( \frac{Z_1 \cdot Z_2 \cdot e^2}{r} - E \right) \right]^{\frac{1}{2}} dr}$$

– Define:

$$\eta = 2\pi \cdot \frac{Z_1 \cdot Z_2 \cdot e^2}{\hbar v}$$

to obtain

far outside classical turning point

= “Sommerfeld Parameter”,

describes s-wave penetration; approximation:

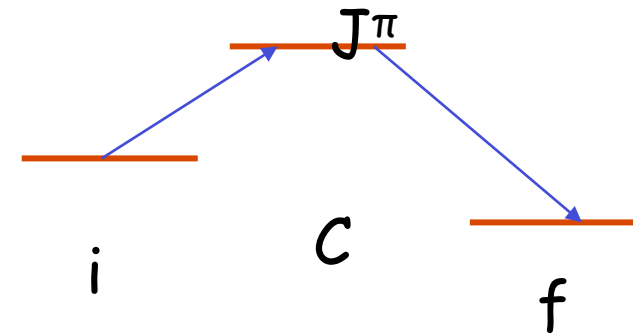
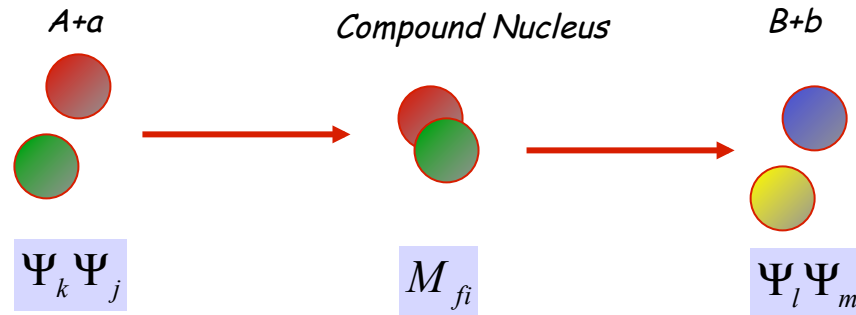
$$P_T = e^{-2\pi\eta}$$

$$2\pi\eta \approx \frac{1}{\sqrt{E}} \cdot 31.28 \cdot Z_1 \cdot Z_2 \cdot \sqrt{A} \cdot [\sqrt{keV}]$$

$$\sigma(E) = \frac{1}{E} \cdot e^{-2\pi\eta} \cdot S(E)$$

**Cross section = geometry \* tunneling \* “astrophysical S-factor”**

# Resonant Reactions



## ☆ Transition Probability i->f:

👉 **Fermi's Golden Rule:**  $\sim |M_{fi}|^2$

- Averaging over Initial States
- Integration over Compound States
- Integration over Final States

👉  $t_{\text{compound}} \sim 10^{-16} \text{sec}$

👉  $t_{\text{kinematics}} \sim r_0/v \sim \text{fm}/0.1c \sim 10^{-22} \text{sec} \ll t_{\text{compound}}$

👉 **C “forgets” initial state**

e.g.:  $^{12}\text{C} + p \rightarrow ^{13}\text{N}^* \rightarrow ^{13}\text{N} + \gamma$

$$\sigma = \sigma_{\text{total}} \cdot g \cdot \left| \langle Bb | H_n | C \rangle \langle C | H_1 | Aa \rangle \right|^2$$

# Nuclear Resonances



## ★ Characteristica

☞ **Position (Energy)**

☞ **Life Time**

☞  $\Delta E \cdot \tau_R = \hbar \rightarrow \Delta E = \Gamma$  **Resonance Width**

☞ **“Decay” through a Resonance**

$$e^{-t/\tau} = \int_{V_{nucleus}} \Psi^* \Psi dV$$



# Resonant Reactions

## ★ Contributions to total Reaction Rate:

👉 **Level Density in Compound Nucleus**

$$\sigma(E) \propto P(E)$$

👉 **Sum of Exit Channels**

$$\Gamma_{total} = \Gamma_p + \Gamma_n + \Gamma_\gamma + \Gamma_\alpha + \Gamma_\beta = \sum_i \Gamma_i$$

👉 **Probability of Forming Compound Nucleus**

~ amplitude of elastic-reaction channel  $A+a \rightarrow C \rightarrow A+a$   $\therefore \Gamma_a$

👉 **Density of States in Spin Space**

– Relative weight of Compound State wrt. Initial State (A+a)

$$w = \frac{2I_C + 1}{(2I_a + 1) \cdot (2I_A + 1)}$$

👉 **Relative Weight of a Specific Exit Channel**

$$\sigma \propto \frac{\Gamma_B}{\Gamma_{total}}$$

# Resonant Reactions

- State Density in Compound Nucleus

➡ Expressed as a Sum of Resonances of Finite Lifetime

$$\Psi(t) = \Psi_0(t_0) \cdot e^{-t/\tau} \cdot e^{-\frac{iE_0 t}{\hbar}}$$

➡ + Fourier Transformation

➡ Normalization  $P=1$

$$\rightarrow P(E) = \frac{\Gamma}{2\pi} \cdot \frac{1}{(E - E_0)^2 + \left(\frac{\Gamma}{2}\right)^2}$$

# Reaction Rates for Resonant Nuclei

★ Therefore, in Combination:

$$\sigma(E) = (2l+1) \cdot \pi \hat{\lambda}^2 \cdot w \cdot \frac{\Gamma_a \Gamma_b}{(E - E_0)^2 + \left(\frac{\Gamma}{2}\right)^2} \quad \text{(Breit-Wigner Formula)}$$



**Geometry \* Spin State Density \* Energy State Density**

$$\Gamma_{\text{tot}} = \Gamma_p + \Gamma_n + \Gamma_\gamma + \Gamma_\alpha + \Gamma_\beta = \sum_i \Gamma_i$$

$\swarrow \quad \swarrow \quad \swarrow$   
 $\sim \text{keV} \quad \sim \text{eV} \quad \sim \text{eV}$

$$\begin{aligned} \Rightarrow \langle \sigma \rangle &= \frac{2}{\pi} \frac{E_0}{(kT)^{3/2}} e^{-E_0/kT} \int_0^\infty \sigma_{\text{res}}(E) dE \\ &= 2\pi^2 \hat{\lambda}^2 (2l+1) w \cdot \frac{\Gamma_a \Gamma_b}{\Gamma} \cdot \frac{1}{kT} e^{-E_0/kT} \\ &= 2.6 \cdot 10^{-3} \cdot (A T_9)^{-3/2} \cdot \int_{\text{Breit-Wigner}} \cdot w \cdot \frac{\Gamma_a \Gamma_b}{\Gamma} \cdot e^{-\frac{E_0}{kT}} \cdot \frac{1}{kT} dE \end{aligned}$$

Approximationen in obiger Ungleichung dienen als für 3 Fälle

- (1)  $E_{\text{res}} \gg E_0$  (weit entfernte Resonanz)
- (2)  $E_{\text{res}} \sim E_0$  (Resonanz bei typischer WW-Energie)
- (3)  $\langle \sigma \rangle = \sum_{i=1}^N \langle \sigma \rangle_{\text{res}, i}$  (hohe Resonanzdichte)

vgl.: v. Stieglitz (1) (2),  
Thomson u. Wind (3)

# Resonant Reactions (comments)

## ★ Narrow Resonances $\Gamma \ll E_0$

$$\langle \sigma \cdot v \rangle = \left( \frac{8}{\pi \cdot \mu} \right)^{1/2} \cdot \left( \frac{1}{kT} \right)^{-3/2} \cdot \int_0^\infty E \cdot \sigma(E) \cdot e^{\frac{-E}{kT}} \cdot dE$$

$$\begin{aligned} \star \quad \langle \sigma \cdot v \rangle &= \left( \frac{1}{\sqrt{\pi}} \right) \cdot \left( \frac{1}{(kT)^{3/2}} \right) \cdot e^{\frac{-E_0}{kT}} \cdot \underbrace{\int_0^\infty \sigma_{res}(E) \cdot dE}_{= 2\pi^2 \tilde{\lambda}^2 (2l+1) \cdot w \cdot \frac{\Gamma_a \cdot \Gamma_b}{\Gamma}} \\ &= 2.6 \cdot 10^{-13} \cdot (AT_7)^{-3/2} \cdot f_{screening} \cdot w \cdot \frac{\Gamma_a \cdot \Gamma_b}{\Gamma} \cdot e^{-\frac{1.16 \cdot E_0}{T_7}} \end{aligned}$$

## ★ Astrophysically-Useful Approximations

- 👉  $E_{res} \gg E_0$  (far-away resonances)
- 👉  $E_{res} \sim E_0$  (resonance at typical interaction energies)
- 👉  $\langle \sigma v \rangle = \sum_{i=1}^N \langle \sigma v \rangle_{res}$  (high density of resonances)

# Characterizing Cosmic Nuclear Reactions



- ... using the “Astrophysical S-Factor”

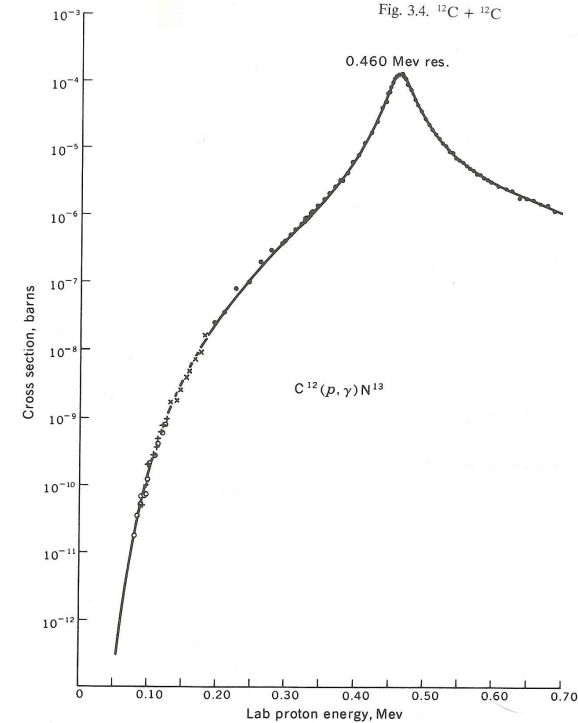
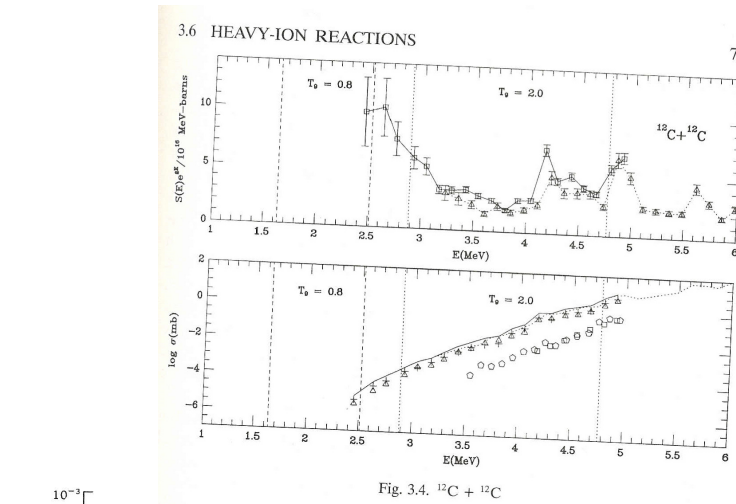
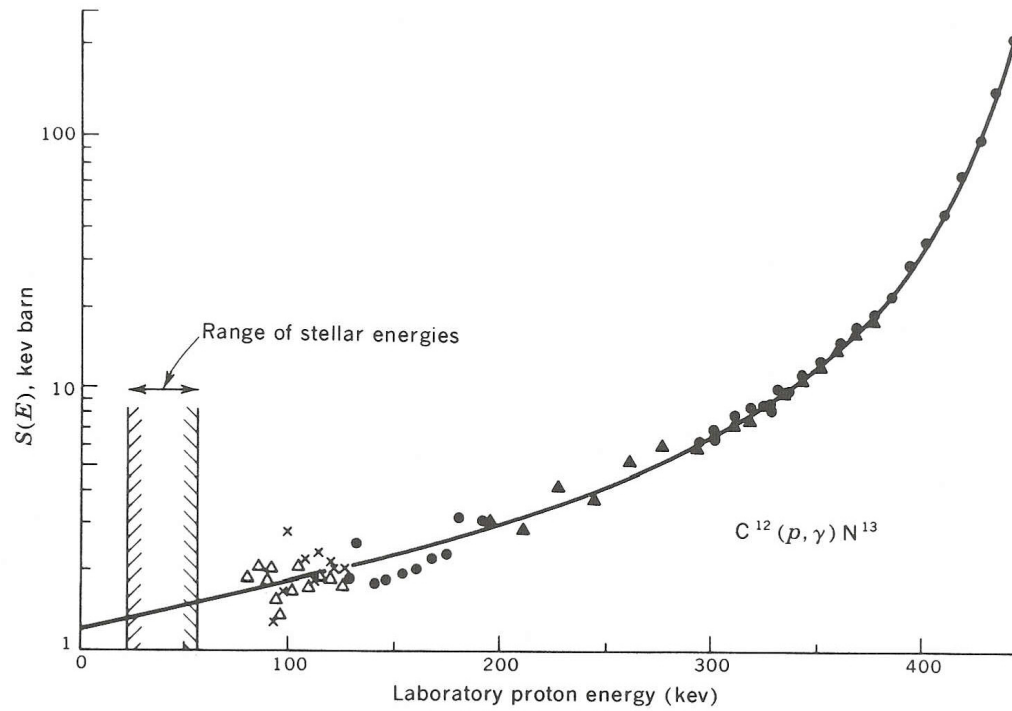


Fig. 4-4 The measured cross section for the reaction  $C^{12}(p, \gamma)N^{13}$  as a function of laboratory proton energy. A four-parameter theoretical curve has been fitted to the experimental points. An extrapolation to  $E_p = 0.025$  MeV, which is an interesting energy for this reaction in astrophysics, appears treacherous. (Courtesy of W. A. Fowler and J. L. Vogt.)

# Reactions for Thermal Populations: Gamov Peak



- Approximating the Reaction Rate Integral

$$\langle \sigma \cdot v \rangle = \left( \frac{8}{\pi \cdot \mu} \right)^{1/2} \cdot \left( \frac{1}{kT} \right)^{3/2} \cdot \int_0^{\infty} E \cdot \sigma(E) \cdot e^{\frac{-E}{kT}} \cdot dE$$

$$\langle \sigma \cdot v \rangle = \left( \frac{8}{\pi \cdot \mu} \right)^{1/2} \cdot \left( \frac{1}{kT} \right)^{3/2} \cdot \langle S(E) \rangle \int_0^{\infty} e^{-\underbrace{\left( \frac{E}{kT} + \frac{b}{\sqrt{E}} \right)}_{=\Phi}} \cdot dE$$

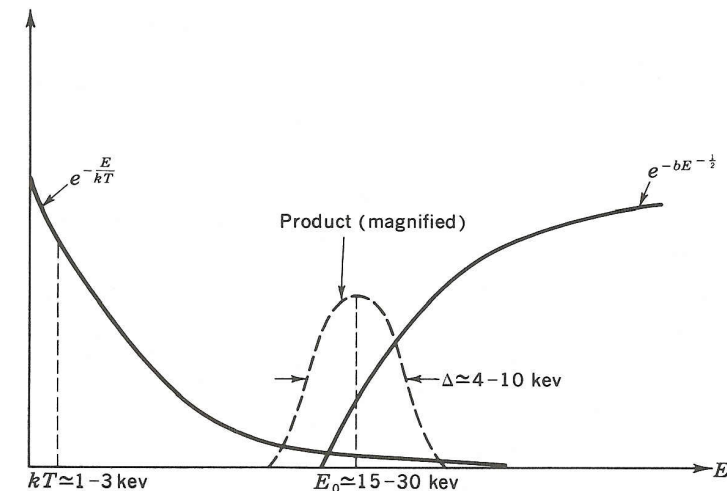
➡ **Maximum at**  $\frac{d\Phi}{dE} = 0 \Rightarrow \frac{1}{kT} - \frac{b}{2} E^{-3/2} = 0$

➡ **=> “Gamov Peak” Energy**

$$E_0 = \left( \frac{bkT}{2} \right)^{2/3}$$

$$E_0 = \left( 15.14 \cdot Z_1 \cdot Z_2 \cdot A^{1/2} \cdot kT \right)^{2/3}$$

$$E_0 = 1.22 \cdot (Z_1^2 Z_2^2 \cdot A \cdot T_6^2)^{1/3} \cdot [keV]$$



**Fig. 4-6** The dominant energy-dependent factors in thermonuclear reactions. Most of the reactions occur in the high-energy tail of the Maxwellian energy distribution, which introduces the rapidly falling factor  $\exp(-E/kT)$ . Penetration through the coulomb barrier introduces the factor  $\exp(-bE^{-1/2})$ , which vanishes strongly at low energy. Their product is a fairly sharp peak near an energy designated by  $E_0$ , which is generally much larger than  $kT$ . The peak is pushed out to this energy by the penetration factor, and it is therefore commonly called the *Gamow peak* in honor of the physicist who first studied the penetration through the coulomb barrier.

# Nuclear-Reaction Network Equations

## ★ How Does the Abundance of a Species $j$ Change?

👉 Sum over Production and Destruction Reactions of all Types

$$\left. \frac{dn_i}{dt} \right|_{\rho=\text{const}} = \sum_j N_j^i r_j + \sum_{j,k} N_{j,k}^i r_{j,k} + \sum_{j,k,l} N_{j,k,l}^i r_{j,k,l}$$

↓
↓
↓

Decays, Photo-Disintegrations,  $e^- / e^+$  Captures, Neutrino Reactions
Two-Particle Reactions  $i+j$ 
Three-Particle Reactions  
(i.e., reaction with a temporary-produced unstable species)

$N_x^i$  Nucleus Accounting Coefficients

$N_j^i = N^i$  number of nuclei  $i$  produced/destroyed by a reaction

$N_{j,k}^i = N^i \prod_{m=1}^{n_{j,k}} |N_m|!$   $N_m$  = number of nuclei of species  $m$  produced/destroyed by a reaction  $i,j,k$   
 There are  $n_{j,k}$  types of species, which are involved in reaction  $i,j,k$   
 and hence need to be summed

## ★ Separate Nuclear-Reaction Composition Changes from Expansion:

👉 Fractional Species Abundance  $Y_i \equiv \frac{n_i}{\rho N_A}$  and hence  $A_i \cdot Y_i$  = mass fraction,  $\sum_i A_i \cdot Y_i = 1$

👉 
$$\frac{dY_i}{dt} = \sum_j N_j^i \lambda_j Y_j + \sum_{j,k} N_{j,k}^i \langle j, k \rangle \cdot \rho N_A \cdot Y_j Y_k + \sum_{j,k,l} N_{j,k,l}^i \cdot \langle j, k, l \rangle (\rho N_A)^2 \cdot Y_j Y_k Y_l$$

# Energy Scaling for Nucleosynthesis

- Typical Energy Scaling for Physical Processes

$$E^2 = (pc)^2 + (mc^2)^2$$

$$E = \sum_{\text{species, particles}} E_{\text{kinetic}} + \sum_{\text{species, particles}} E_{\text{rest\_mass}} = E_{\text{plasma, internal}}$$

👉  **$E_{\text{rest mass}} = \text{const. for all possible states and cases.} \rightarrow \text{Re-normalize:}$**

$$E' = E - \sum_{\text{species, particles}} E_{\text{rest\_mass}} = \sum_{\text{species, particles}} E_{\text{kinetic}} = E'_{\text{plasma, internal}}$$

- This Cannot Be Done for Nucleosynthesis Processes!

👉 **More Precisely, Therefore:**

$$E = \frac{1}{V} \cdot \int_0^\infty \frac{p^2}{2m} \cdot \frac{dN}{dp} dp + \sum_i N_A \cdot m_i c^2 \cdot Y_i$$

$\underbrace{\hspace{1.5cm}}_{E_{\text{kin}}} \quad \underbrace{\hspace{1.5cm}}_{E_{\text{rest}}}$

**and then Coarsely Renormalize:**  $\epsilon_{\text{rest\_mass}} \approx \frac{1 \text{amu}}{\text{nucleon}}$

$$E \rightarrow E' = E - N_A m_{\text{amu}} c^2$$

👉 **Then, also:**

$$E' = E_{\text{kin}} + -N_A c^2 \sum_i \Delta M_i Y_i$$

mass excess

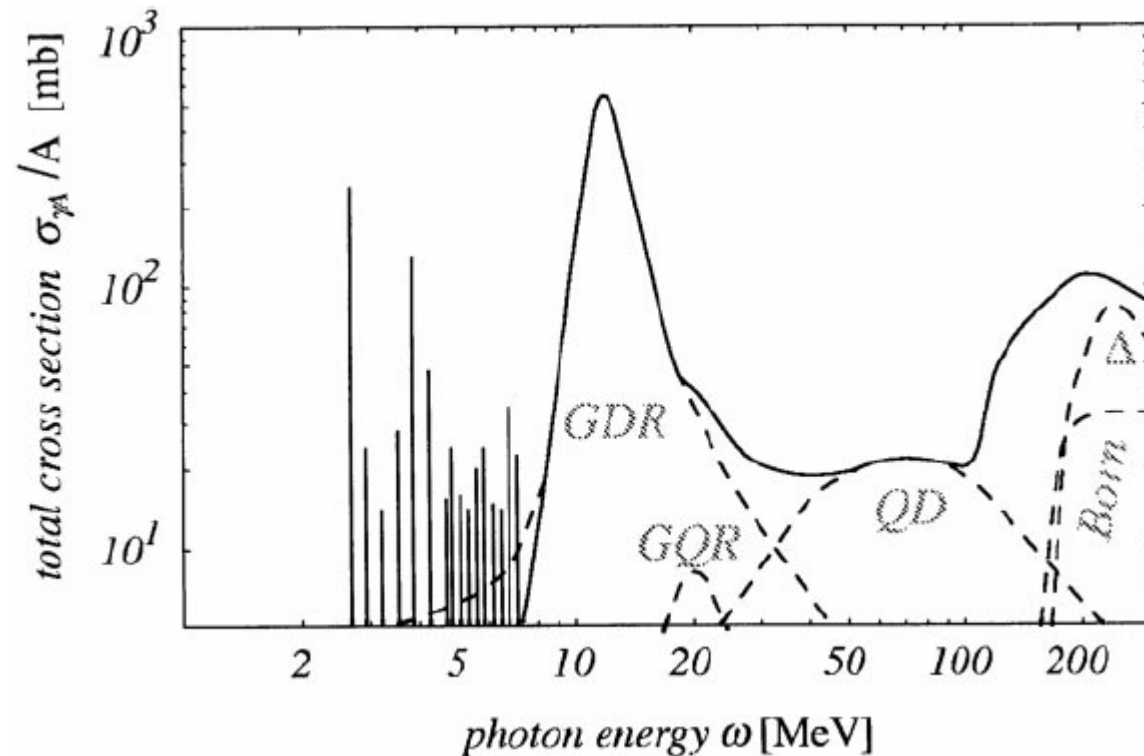
$$E' = E_{\text{kin}} + N_A c^2 \left[ Y_e (m_p - m_{\text{amu}}) + (1 - Y_e) (m_n - m_{\text{amu}}) + \sum_i B_i Y_i \right]$$

binding energy

$$E' \rightarrow E \text{ from here onward}$$



# States of Nuclei and their Components



★ **We See Effects of** (with increasing energy):

- ☞ **Excitation of Single Nucleons in Nucleus Potential ("Nuclear Lines")**
  - $h\nu = E_{\text{nucl}}$
- ☞ **Collective Excitations of Nucleon Groups ("Pygmi/Giant Resonances")**
  - giant resonances: protons versus neutrons
  - quasi-deuteron resonances: a pair of proton and neutron
  - each of these occur in all multipole orders
- ☞ **Excitations of Single Nucleons ("Delta Resonance")**
- ☞ **... → Hadron/Quark Phase Transitions**

# ★ Particle-Antiparticle Annihilation

# Pair Production Processes

## ★ Energy Threshold:

☞  $E = mc_e^2$       1.022 MeV

## ★ Processes:

–  $\gamma + \nu \rightarrow e^+ + e^-$       (dominant)

and

–  $\gamma + e^+/e^- \rightarrow e^+/e^- + e^+ + e^-$

–  $\gamma + Z \rightarrow Z + e^+ + e^-$

–  $e^+/e^- + e^+/e^- \rightarrow e^+/e^- + e^+/e^- + e^+ + e^-$

–  $e^+/e^- + Z \rightarrow Z + e^+/e^- + e^+ + e^-$

☞ Processes with photons scale  $\sim a s_T$ ,

☞ Processes with electrons scale  $\sim a^2 s_T$

## ★ PP Cross Section:

$$\sigma_{\gamma\gamma}(s) = \frac{3}{8} \frac{\sigma_T}{s} \left[ \left( 2 + \frac{2}{s} - \frac{1}{s^2} \right) \cosh^{-1} s^{1/2} \right]$$

$$s = \left( \frac{E}{m_e c^2} \right)^2$$

$$\sigma_{\gamma\gamma}(s) = \frac{3}{8} \frac{\sigma_T}{s} \sqrt{s-1} \quad s-1 \ll 1$$

$$\sigma_{\gamma\gamma}(s) = \frac{3}{8} \frac{\sigma_T}{s} \ln(4s) - 1 \quad s \gg 1$$

# Pair Annihilation Processes

★ Analogous to Pair Productions (Detailed Balance)

★ Processes:



★ Annihilation Cross Section:

👉 limiting cases ( $e^\pm$  slow/relativistic in center of mass frame) →

$$\sigma_{ann} = \frac{3}{16} \sigma_T \left[ \left( 1 + \beta_{cm}^2 \right) / \beta_{cm} \right] \forall (\beta_{cm} \ll 1)$$

$$\sigma_{ann} = \frac{3}{16} \sigma_T \left[ \left( 2 \ln(2\gamma_{cm}) - 1 \right) / \gamma_{cm} \right] \forall (\beta_{cm} \gg 1)$$

★ Cooling Rate for Plasma due to Annihilation:

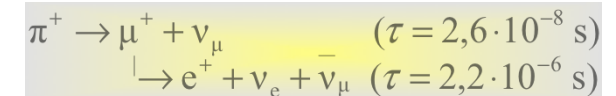
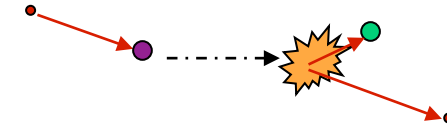
$$\Lambda_{ann} = (2m_e c^2) \sigma_{ann} = \frac{3}{4} \sigma_T m_e c^3$$

# Positron Production Processes

## ✓ Cosmic-Ray Nuclear Reactions

☆ e.g.  $^{12}\text{C}(p,pn)^{11}\text{C}(\beta^+)$ , or  $^{16}\text{O}(p,\alpha)^{13}\text{N}(\beta^+)$

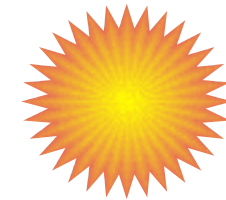
☆ Pion Production in HE Collisions



## ✓ Hot-Plasma Pair Production

☆ 'kT>MeV'-Plasma

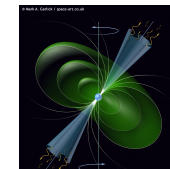
- ☞ Accretion Columns & Disks
- ☞ Jet Bases



## ✓ E.M.-Cascade Pair Production

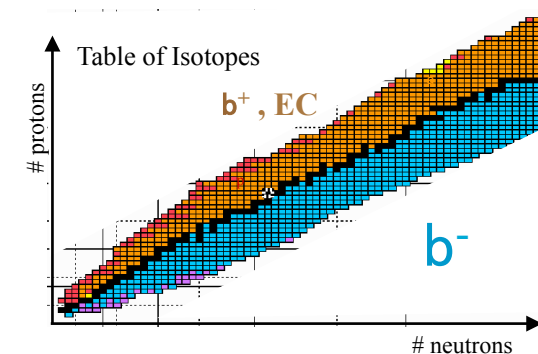
☆ Strong Magnetic Fields

- ☞ Pulsars
- ☞ Jets



## ✓ Nucleosynthesis

☆ e.g.  $^{56}\text{Ni}(\beta^+)$ ,  $^{44}\text{Ti}(\beta^+)$ ,  $^{26}\text{Al}(\beta^+)$ ,  $^{22}\text{Na}(\beta^+)$ ,  
 $^{13}\text{N}(\beta^+)$ ,  $^{14}\text{O}(\beta^+)$ ,  $^{15}\text{O}(\beta^+)$ ,  $^{18}\text{F}(\beta^+)$

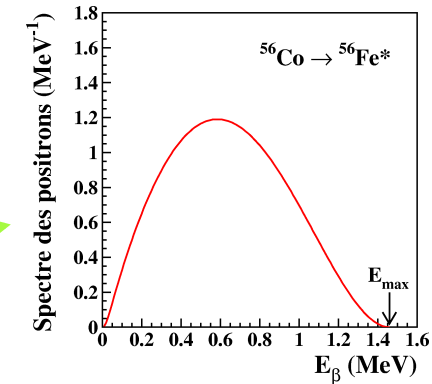


# Known Sources of Positrons, $E_{e^+}$

- Radioactive Nuclei

- ★ Sources:

Supernovae, Novae,  
Cosmic Rays & ISM  
~MeV



- ★ Positron Energies:

- Pion Production

- ★ Sources:

Cosmic Rays & ISM

- ★ Positron Energies:

$\langle E \rangle \sim 30$  MeV (MeV...GeV)

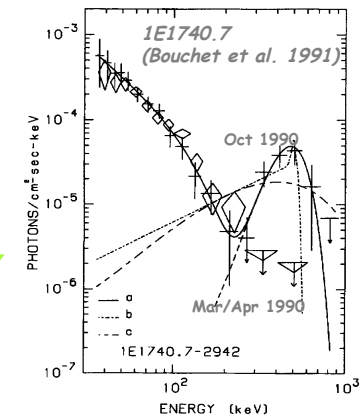
- Pairs from Hot Plasma

- ★ Sources:

Accreting Binaries

- ★ Positron Energies:

~MeV  $T > 100$  keV ( $E_{thr} = 1.02$  MeV)



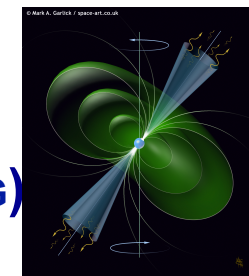
- Pairs from Strong Magnetic Fields

- ★ Sources:

Pulsars, Magnetars

- ★ Positron Energies:

~MeV...GeV ( $E_{thr} = 1.02$  MeV  $B > 10^{12}$  G)



# Positron Annihilation



## ★ Charge Exchange with H Atoms

- ☞ Coulomb Attraction,  
 $p + e^- + e^+ \rightarrow \text{Ps} + p$
- ☞ Most Efficient <50 eV
- ☞ Threshold Energy 6.8 eV
- ☞ In Neutral H, After Slowing-Down by Ionization and Excitation

## ★ Radiative Capture of Free $e^-$

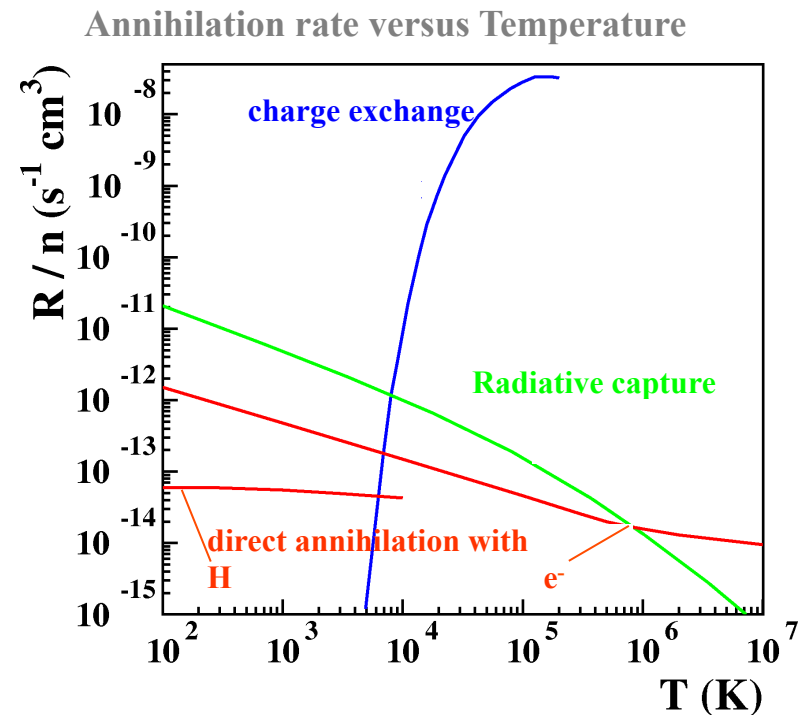
- ☞ Momentum Balance:  
 $e^- + e^+ \rightarrow \text{Ps} + h\nu$
- ☞ In (partly-)ionized ISM,  
After Thermalization by Coulomb Interactions

## ★ Direct Annihilation

- ☞  $e^- + e^+ \rightarrow 2h\nu$  (511 keV)

## ★ Annihilation Rate Depends on Phase

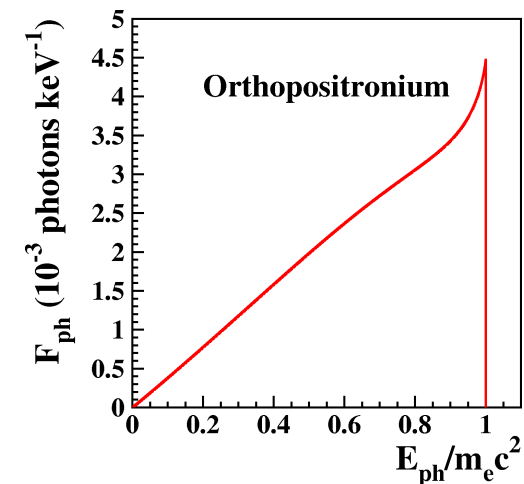
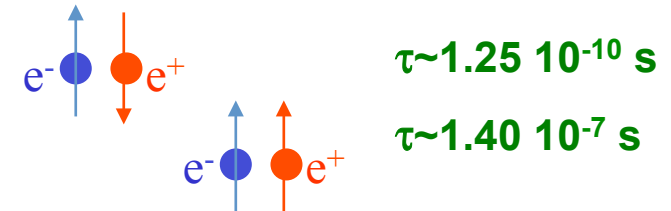
- ☞  $10^3 \dots 10^5 \dots 10^8 \text{ y}$



# Positronium



- “Atom” with  $e^-$  and  $e^+$
- Relative Spin Orientations
  - ☆ Singlet State  $^1S_0$ / Para-Positronium
  - ☆ Triplet State  $^3S_1$ / Ortho-Positronium
- Spin & Momentum Balances
  - ☆ 2-Photon Annihilation Only for Para-Ps:
    - $2 \gamma$  at  $\sim 511 \text{ keV}$
    - $\leftarrow s=1 \rightarrow s=1$
  - ☆ 3-Photon Annihilation from Ortho-Ps
  - ☆ Annihilation on Grains:  $2\gamma$  also for Ortho-Ps
- Annihilation Spectrum
  - ☆ Line/Continuum  $\gamma$  Ratio 1.45
  - ☆ 2.75 Annihilation  $\gamma$ 's per  $e^+$

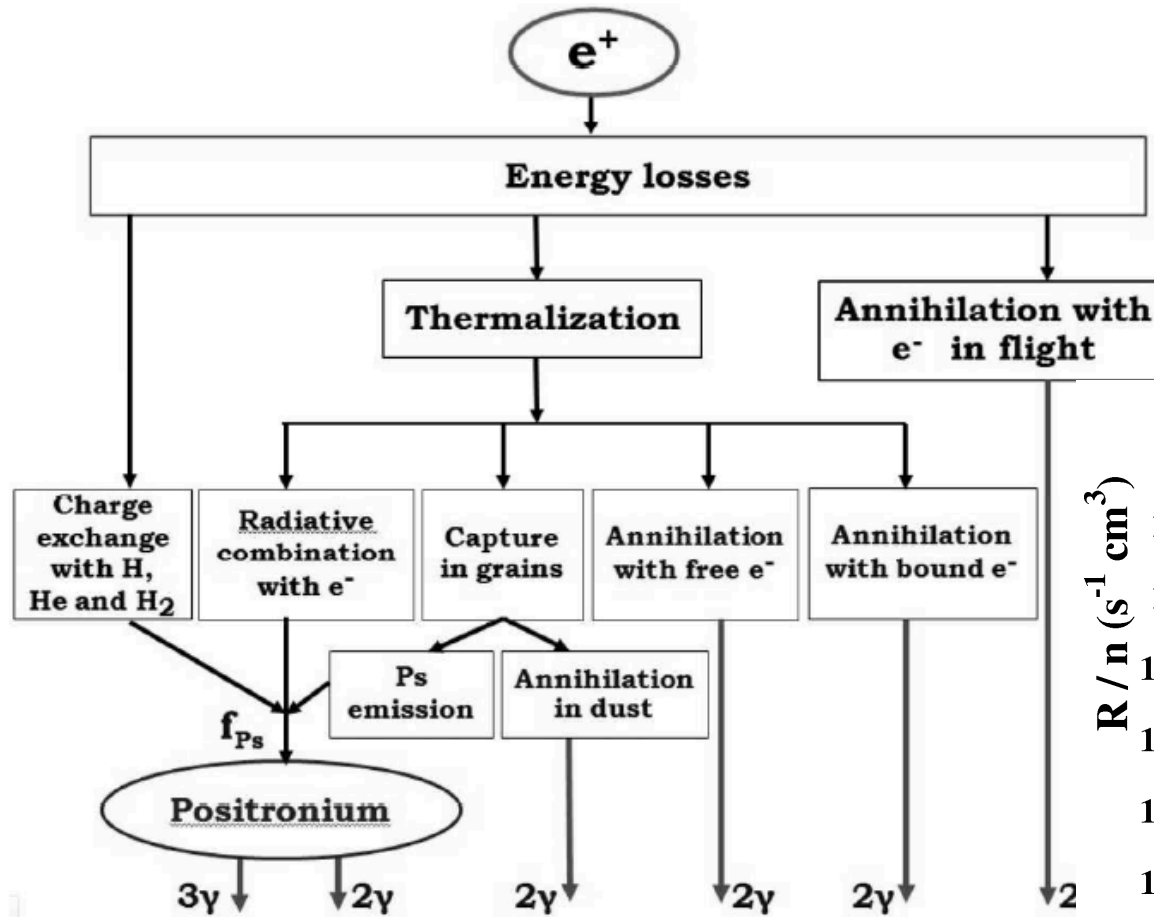




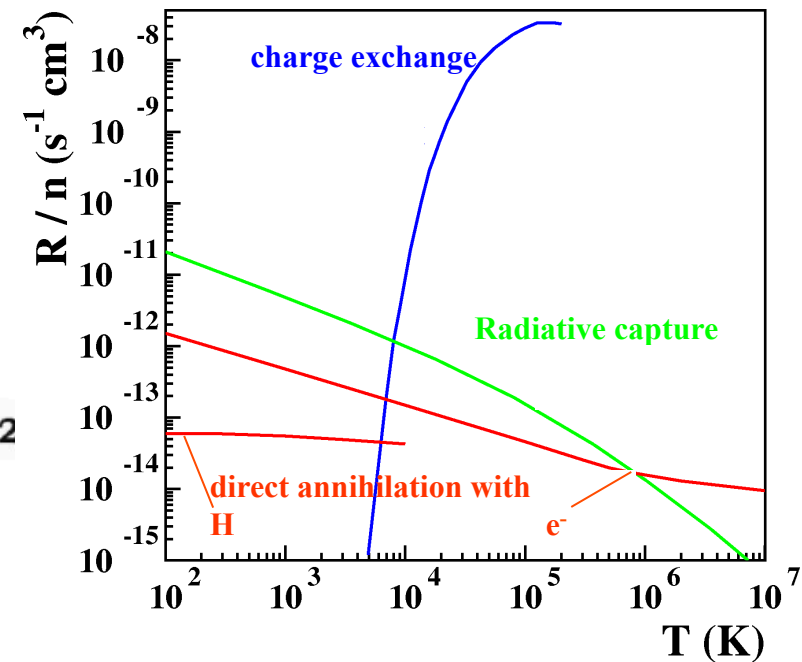
# Annihilation and Ambient-Gas Conditions



- ★ Variety of Channels to 'Capture' an  $e^-$
- ★ Dominating: Charge Exchange with H Atoms



Annihilation rate versus Temperature



# The Sources of Positrons: Which $E_{e^+}$ ?

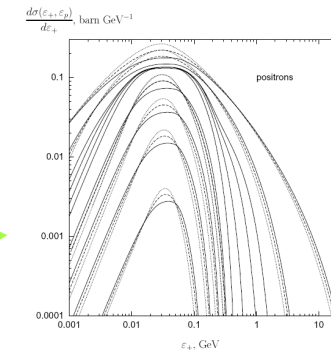
- Pion Production

☆ Sources:

☆ Positron Energies:

Cosmic Rays & ISM

$\langle E \rangle \sim 30$  MeV



- Pairs from Hot Plasma

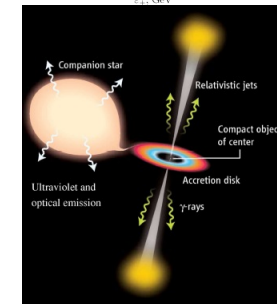
☆ Sources:

☆ Positron Energies:

$T > 100$  keV ( $E_{thr} = 1.02$  MeV)

Accreting Binaries

$\sim$  MeV



- Pairs from Strong Magnetic Fields

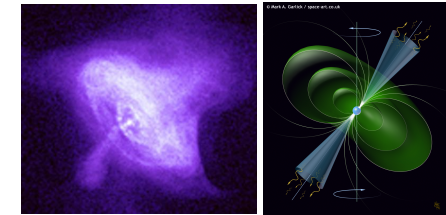
☆ Sources:

☆ Positron Energies:

( $E_{thr} = 1.02$  MeV) ( $B > 10^{12}$  G)

Pulsars, Magnetars

$\sim$  MeV...GeV



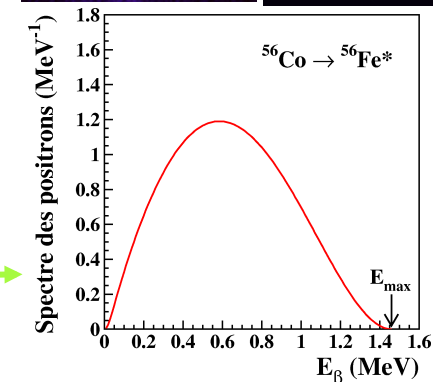
- Radioactive Nuclei

☆ Sources:

☆ Positron Energies:

Supernovae, Novae,  
Cosmic Rays & ISM

$\sim$  MeV



# Positron Propagation



☆ from GeV...MeV Energies to ~eV

- Interactions with ISM Gas

☆ Ionization ( $H_I$ ,  $H_2$ )

☆ Coulomb Scattering ( $H_{II}$ )

☆ Bremsstrahlung

👉 **Slowing-Down Time**

$\sim 10^3 \dots 10^5 \dots 10^7$  y

for  $MC / n_H \sim 1 \text{ cm}^{-3} / \text{hot ISM}$

$$\left( = \int_{E_{th}}^{E_i} \frac{dE}{(dE/dt)} \right)$$

- Range

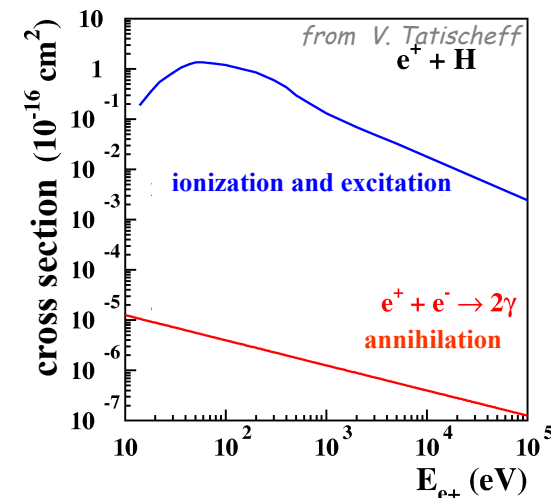
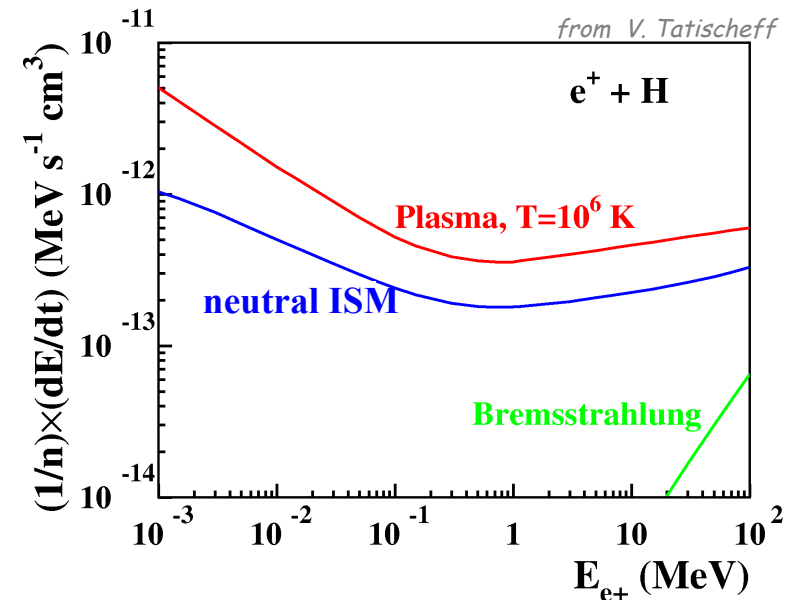
👉 **Larmor Radius**  $\sim 3 \cdot 10^{-10} \text{ pc}$  (5mG, 1MeV)

👉 **Diffusion in Turbulent/Hot ISM**

$\sim 800 \text{ pc}$  (Bykov & Fleishmann '92)

- Annihilation

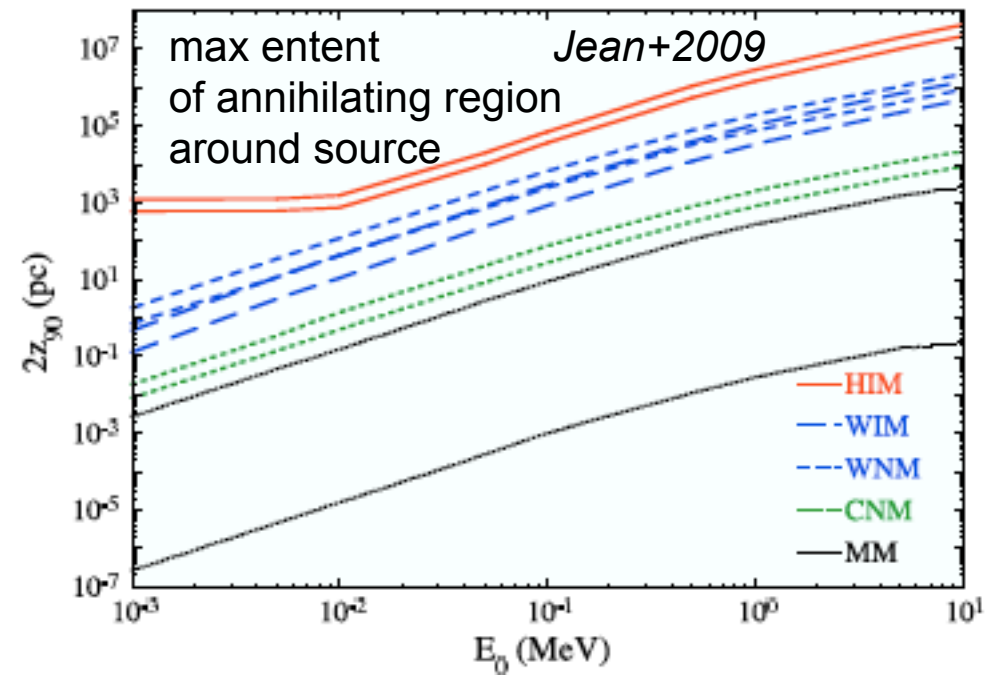
☆ **Annihilation In-Flight** ~rare



# Positron Propagation Away from Sources



- ★ Positrons May Escape Source Regions with Hot/Tenuous Surroundings
- ★ Magnetic-Field Structure is Crucial



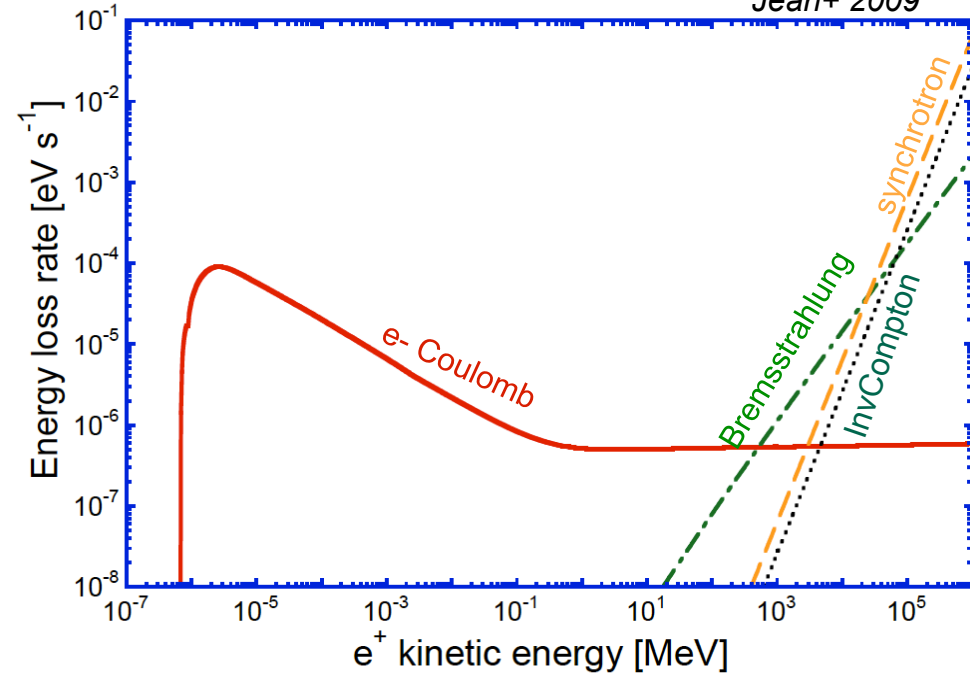
# Relativistic- $e^+$ Interactions with Gas

Jean+ 2009



## ★ Slowing Down

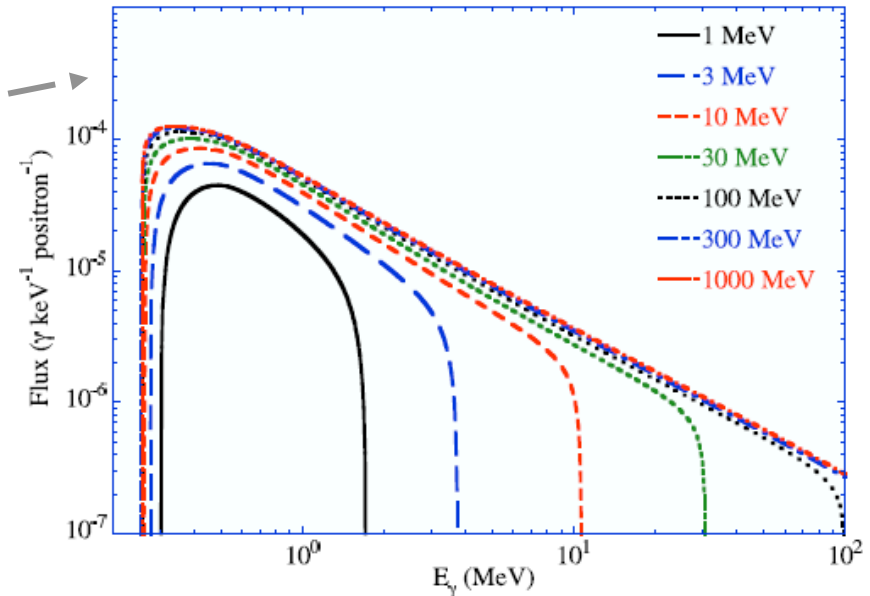
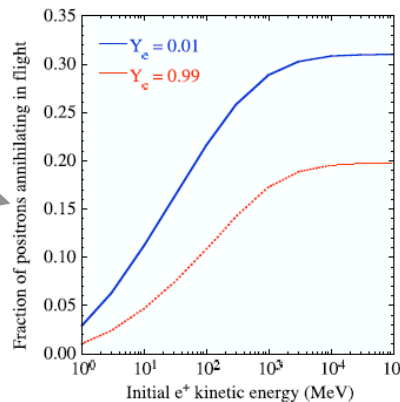
- ☞ Ionization
- ☞ Coulomb Collisions
- ☞ Bremsstrahlung
- ☞ Inverse-Compton Scattering
- ☞ Synchrotron Emission
- ☞ Slowing Down Time May Be
  - $\tau \sim 10^3 \dots 10^5 \dots 10^7$  yrs



## ★ Annihilation

- ☞ 'Direct' / 'In-Flight' with Ambient  $e^-$  (or H)
- ☞ Doppler-Shifted Annihilation Line
- ☞ Probability  $\sim 10\%$  (MeV)

» E loss processes



# Annihilation Conditions: Which ISM Phase



## ★ Diversity of Annihilation Processes:

- Direct Annihilation with Free or Bound e-
- Formation of a Positronium Atom
- At MeV Energies
- After Slowing Down
- On Surfaces of Dust

👉 Momentum Balance  $\leftrightarrow$  Line Width

